Large N gauge theories with adjoint fermions

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The single site model

Gauge action: $S_g = -12bNP; \quad P = \frac{1}{12}\sum_{\mu\neq\nu=1}^{4} \mathrm{Tr}\left[U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger\right]$

Fermion action:

$$S_f = -f \ln\det H_o(\mu)$$

$$H_o(\mu) = \frac{1}{2}\left[(1+\mu)\,\gamma_5 + (1-\mu)\,\epsilon(H)\right]$$

H is the massive Hermitian Wilson Dirac operator

$b = \frac{1}{g^2 N}$ is the inverse lattice ‘t Hooft coupling

μ is the fermion mass

f is the number of Dirac flavors

We can ignore gauge field topology in the large N limit and assume all gauge fields are in the zero topological sector

Due to the chiral symmetry obeyed by the overlap-Dirac operator, the eigenvalues of $H_o(0)$ come in doubly degenerate positive-negative pairs.

The determinant is positive definite and we can set f to any real value

Weak Coupling Perturbation Theory

Polyakov loop eigenvalues

$$D_\mu^{ij} = e^{i\theta_\mu^i}\delta_{ij}$$

Perturbation

$$U_\mu = e^{ia_\mu} D_\mu e^{-ia_\mu}$$

Leading order result

$$S_g = \sum_{i\neq j} \ln\left[\sum_\mu \sin^2\frac{1}{2}\left(\theta_\mu^i - \theta_\mu^j\right)\right]$$

$$S_f = -2f\sum_{i,j}\ln\left[\frac{1+\mu^2}{2} + \frac{1-\mu^2}{2}\frac{2\sum_\mu \sin^2\frac{\theta_\mu^i-\theta_\mu^j}{2} - m}{\sqrt{\left(2\sum_\mu \sin^2\frac{\theta_\mu^i-\theta_\mu^j}{2} - m\right)^2 + \sum_\mu \sin^2\left(\theta_\mu^i - \theta_\mu^j\right)}}\right]$$

If all $\theta_\mu^i = 0$ then $S_g \to -\infty$ But, if $\mu=0$ then $S_f \to \infty$

Single site Polyakov loop eigenvalues and momenta on the infinite lattice

At lowest order in weak coupling perturbation theory
adjoint fermions on a single site lattice see momentum modes

$$e^{i(\theta^i_\mu - \theta^j_\mu)} \quad 1 \le i,j \le N$$

The N^2-1 angles, $\theta^i_\mu - \theta^j_\mu$, approach a continuum of momenta, p_μ, as N approaches infinity

We want the measure to be $\int \prod_\mu dp_\mu$ in order to reproduce correct infinite volume perturbation theory

Why do we need overlap fermions on a single site lattice?

<u>Naive fermions</u>

$$S_g = \sum_{i \ne j} \ln\left[\sum_\mu \sin^2 \frac{1}{2}\left(\theta^i_\mu - \theta^j_\mu\right)\right] \qquad S_f = -2f \sum_{i,j} \ln\left[\mu^2 + \sum_\mu \sin^2\left(\theta^i_\mu - \theta^j_\mu\right)\right]$$

Momenta, $p_\mu \in [\frac{\pi}{2}, \pi]$, will spoil the uniform measure in the large N limit.

Massless overlap fermions

$$S_f = -2f \sum_{i \neq j} \ln \left[\frac{1}{2} + \frac{1}{2} \frac{2\sum_\mu \sin^2 \frac{\theta^i_\mu - \theta^j_\mu}{2} - m}{\sqrt{\left(2\sum_\mu \sin^2 \frac{\theta^i_\mu - \theta^j_\mu}{2} - m\right)^2 + \sum_\mu \sin^2\left(\theta^i_\mu - \theta^j_\mu\right)}} \right]$$

Unlike naive fermions, momenta $p_\mu \in [0, \frac{\pi}{2}]$ and $p_\mu \in [\frac{\pi}{2}, \pi]$ are not identified

We need $m > 2\sum_\mu \sin^2 \frac{\theta^i_\mu - \theta^j_\mu}{2}$ for the mode corresponding to that momentum to be massless

$m \to \infty$ is the naive fermion limit and so we cannot make it too large

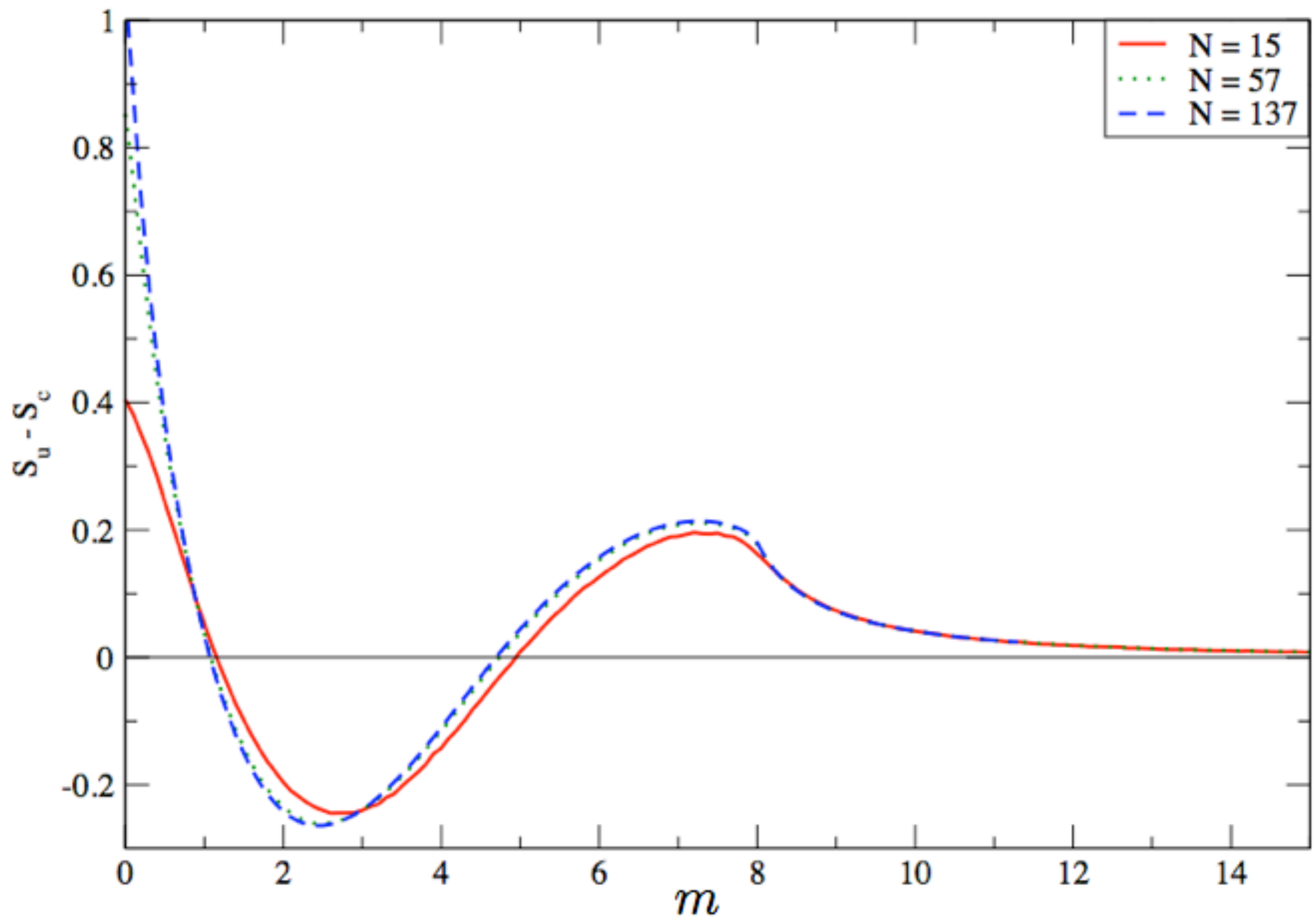
Making it too small will restrict the region inside the Brillouin zone, $p_\mu \in [-\pi, \pi]$

where we have massless fermions and therefore the correct momentum measure

Correlated versus uncorrelated momenta

Correlated momenta $\theta_\mu^j = \frac{2\pi j}{N}$ $j = 1 \cdots, N$ in all four directions S_c: action

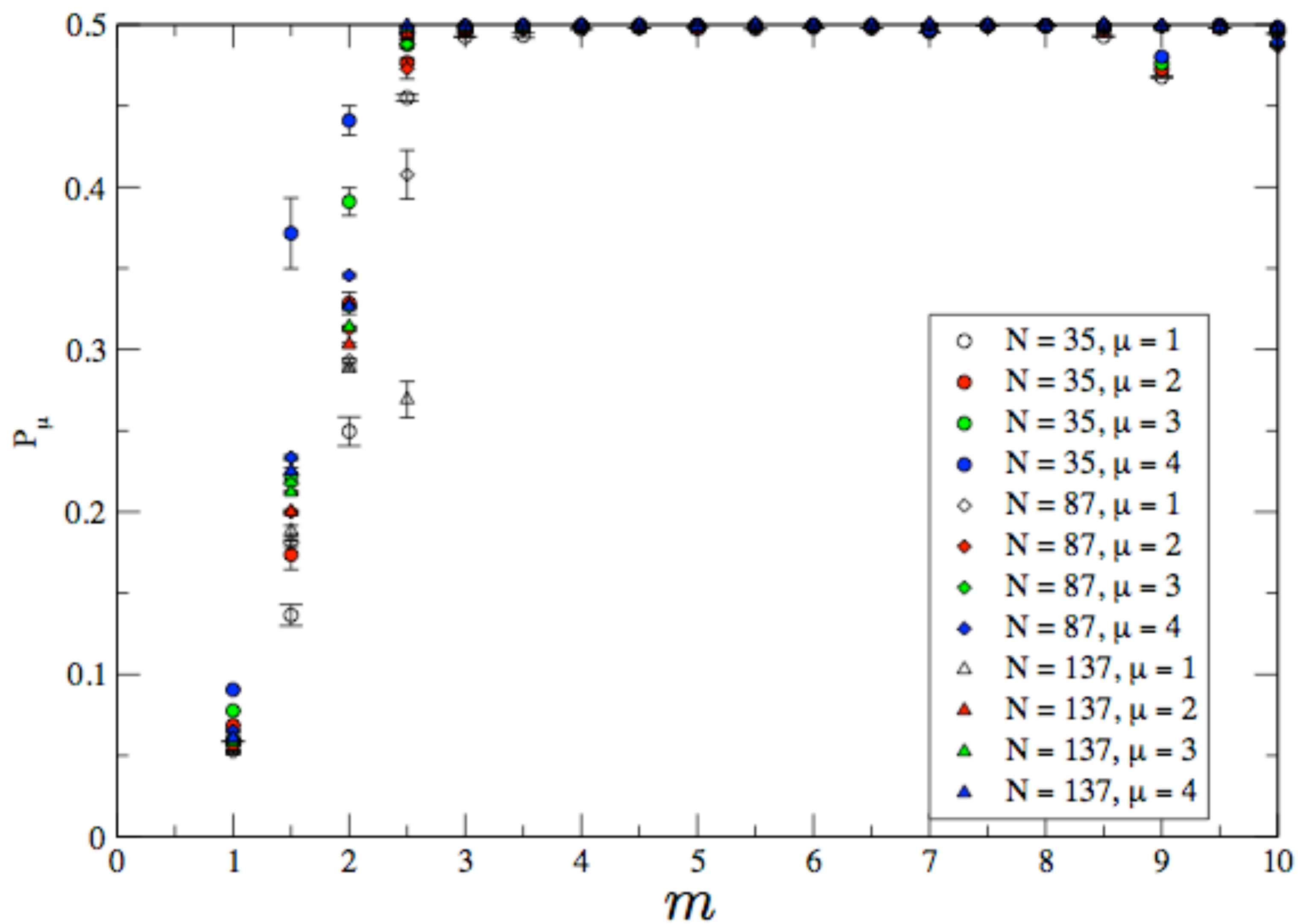
Uncorrelated momenta $\theta_\mu^j = \frac{2\pi \pi_j^\mu}{N}$ π^μ is a permutation of $j = 1 \cdots, N$ S_u: action



$m \in [3.5, 4.5]$
is a good choice

Distribution of Polyakov loop eigenvalues

$$P_\mu = \frac{1}{2}\left(1 - \frac{1}{N^2}|\mathrm{Tr}U_\mu|^2\right) = \frac{1}{N^2}\sum_{i,j}\sin^2\frac{1}{2}\left(\theta^i_\mu - \theta^j_\mu\right)$$



A numerical proposal

- Use Hybrid Monte Carlo Algorithm with Pseudo-fermions.
  - Works for integer number of Dirac flavors.
- A direct fermion HMC algorithm for non-integer Dirac flavors.
- Pick N and b such that we are in the large N limit for that b.
  - We expect N to increase as b increases.
- Pick a quantity to set the scale.
  - Lowest positive eigenvalue of the overlap Dirac operator.
  - Strong to weak coupling transition.
- Find the region of b where we observe scaling.
- Measure physically interesting quantities.

We will report on progress toward this goal by
showing some results with

$$b = \frac{1}{\lambda} = \frac{1}{g^2 N} = \frac{\beta}{2N^2}$$

single Dirac flavor
massless fermions
N=18
b from 0.32 to 0.70

$b = 0.35$

$\beta = 2.8, N = 2$

$\beta = 6.3, N = 3$

beta function

Let t = ln a(b) be a logarithmic scale where a(b) could be the square root of the string tension at the lattice coupling, b.

beta function: $$\beta(b) = \frac{db(t)}{dt}$$

Perturbation theory: $$\beta(b) = -b_0 - \frac{b_1}{b} + \cdots$$

$$b_0 = \frac{11-4f}{24\pi^2}; \quad b_1 = \frac{17-16f}{192\pi^4}$$

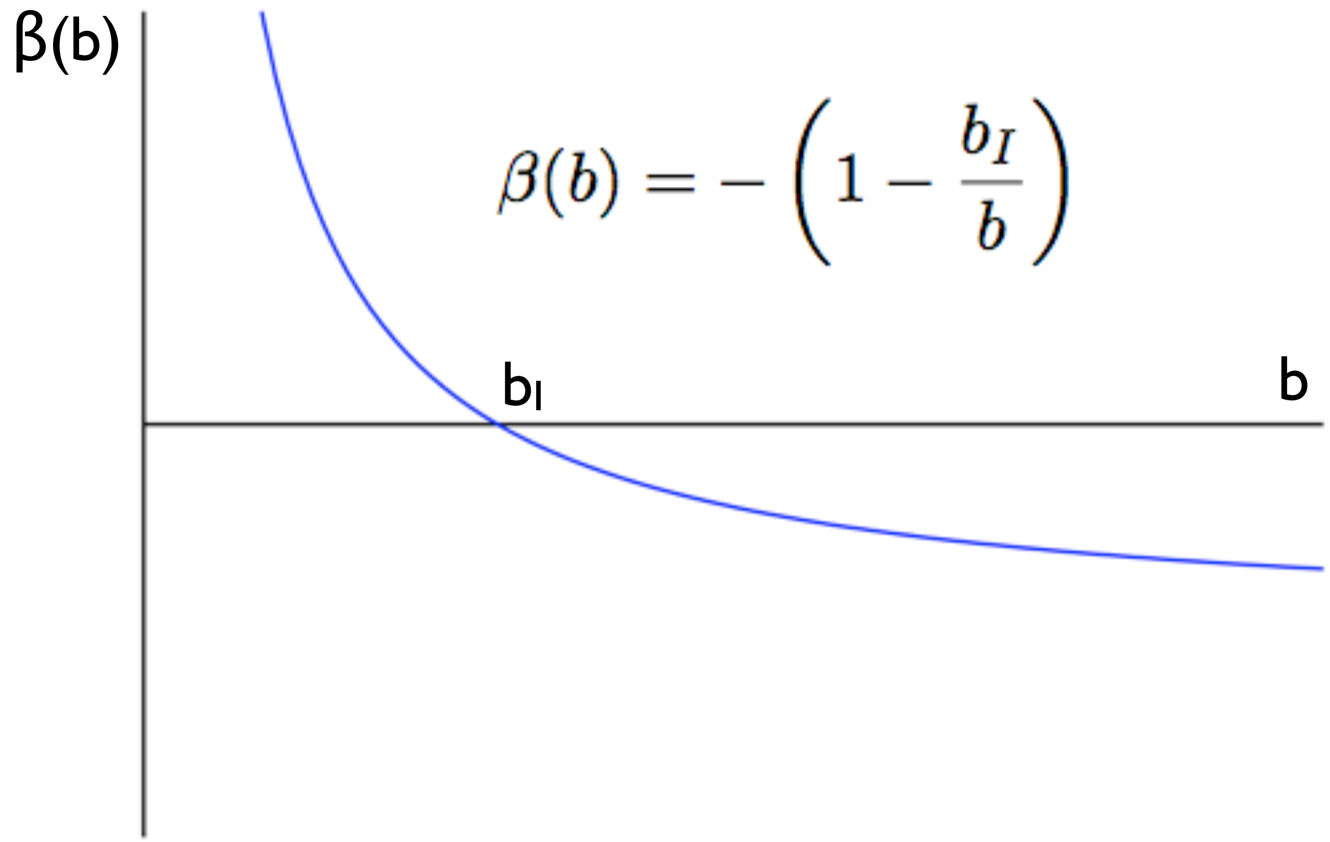
Higher order coefficients depend on the choice of a(b) and the form of the lattice action

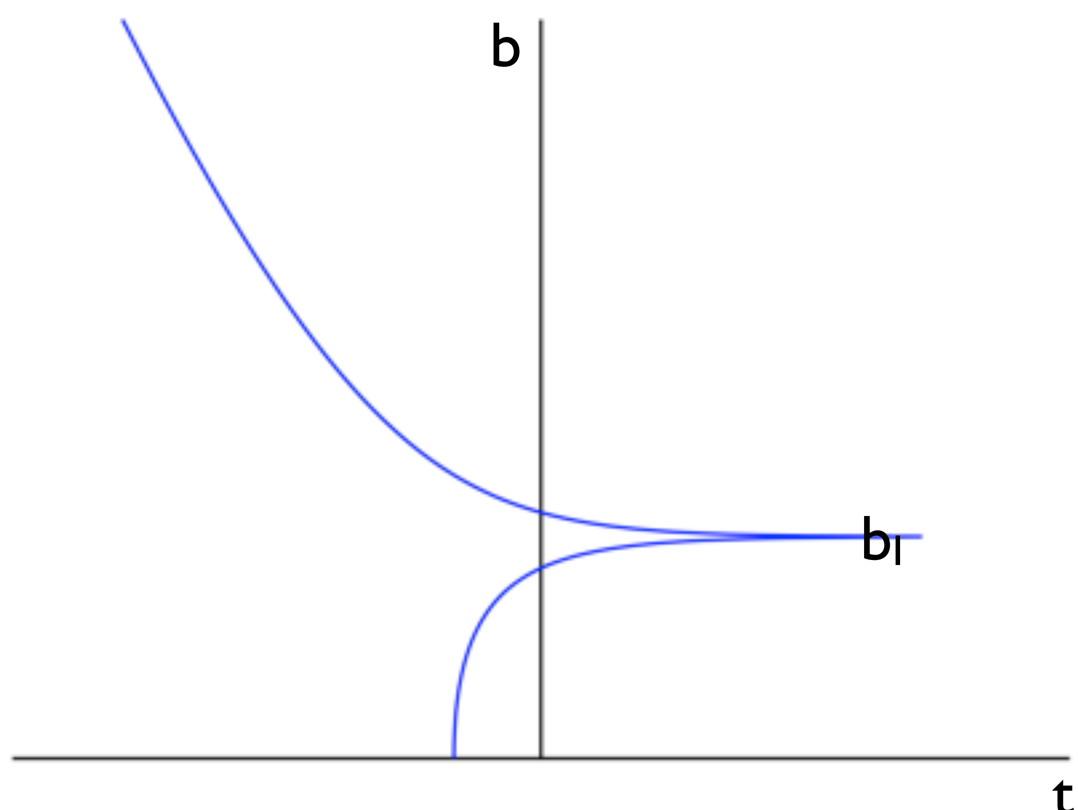
The perturbative beta function has a zero at $$b_*(f) = -\frac{b_1}{b_0} = \frac{1}{8\pi^2}\frac{16f-17}{11-4f}.$$ if $\frac{11}{4} > f > \frac{17}{16}$

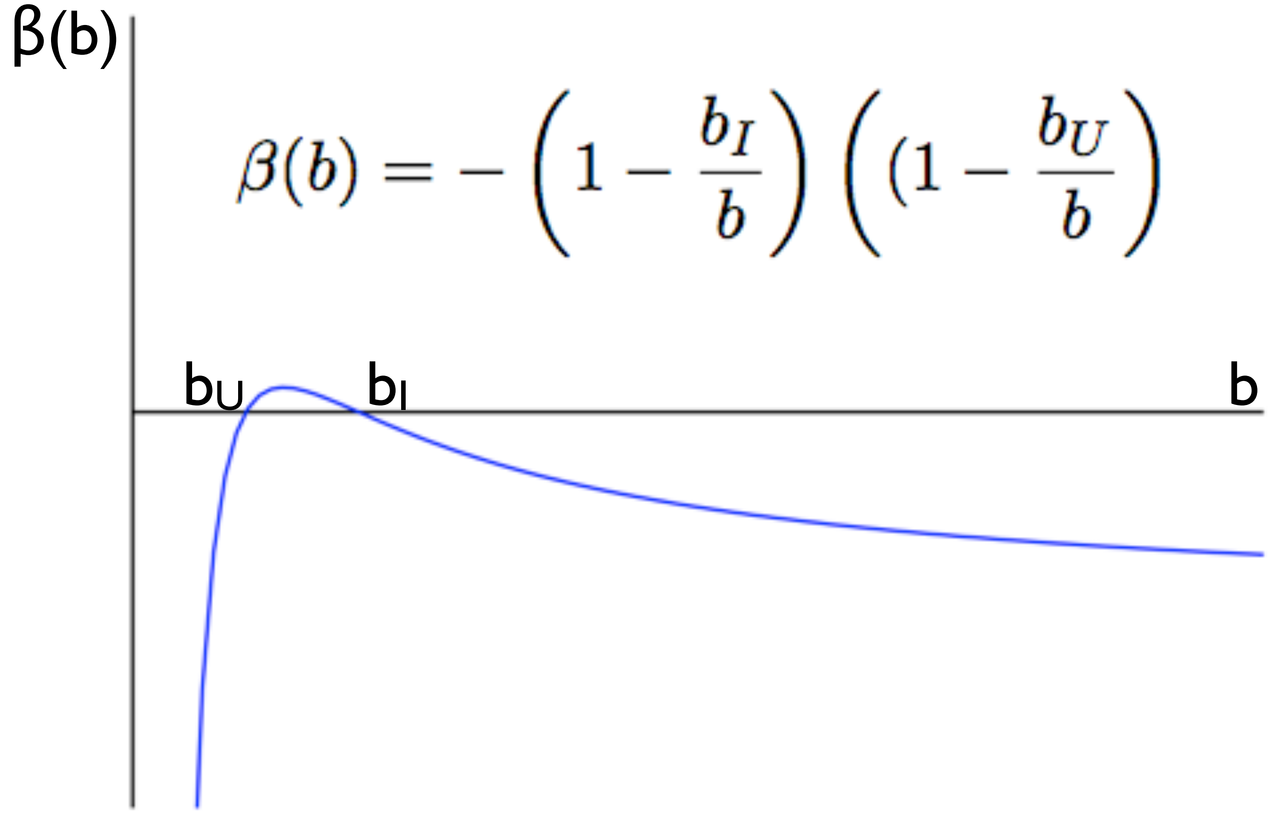
If f is close to 11/4, the zero occurs at very weak coupling and we can expect the zero in perturbation theory to remain essentially unaltered.

If f is close to 17/16, the zero occurs at very strong coupling and it is quite possible that it depends on the choice of a(b) and the form of the lattice action

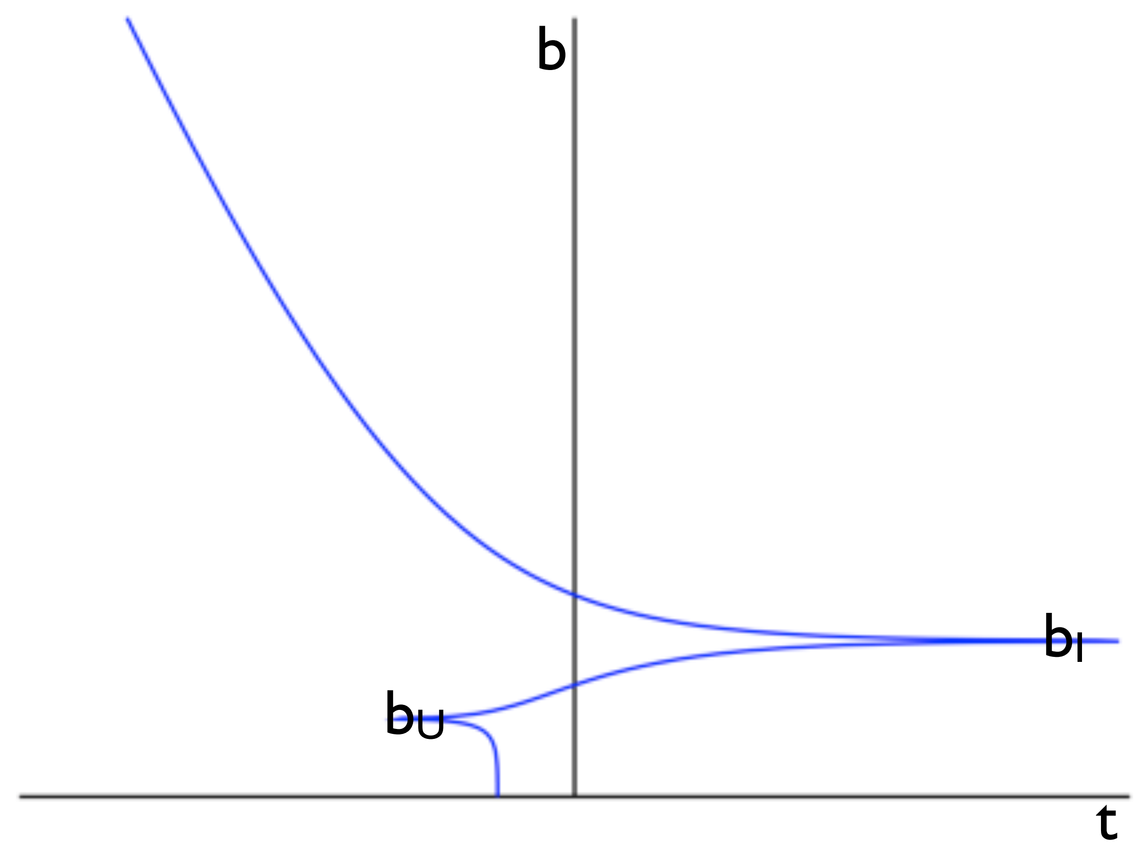
Since a zero of the beta function could be an infra-red or ultra-violet fixed point, we need to study the behavior of the coupling close to this zero in order to see if we can define a continuum theory at the location of this zero.


β(b)
b
b_I
$\beta(b) = -\left(1 - \frac{b_I}{b}\right)$

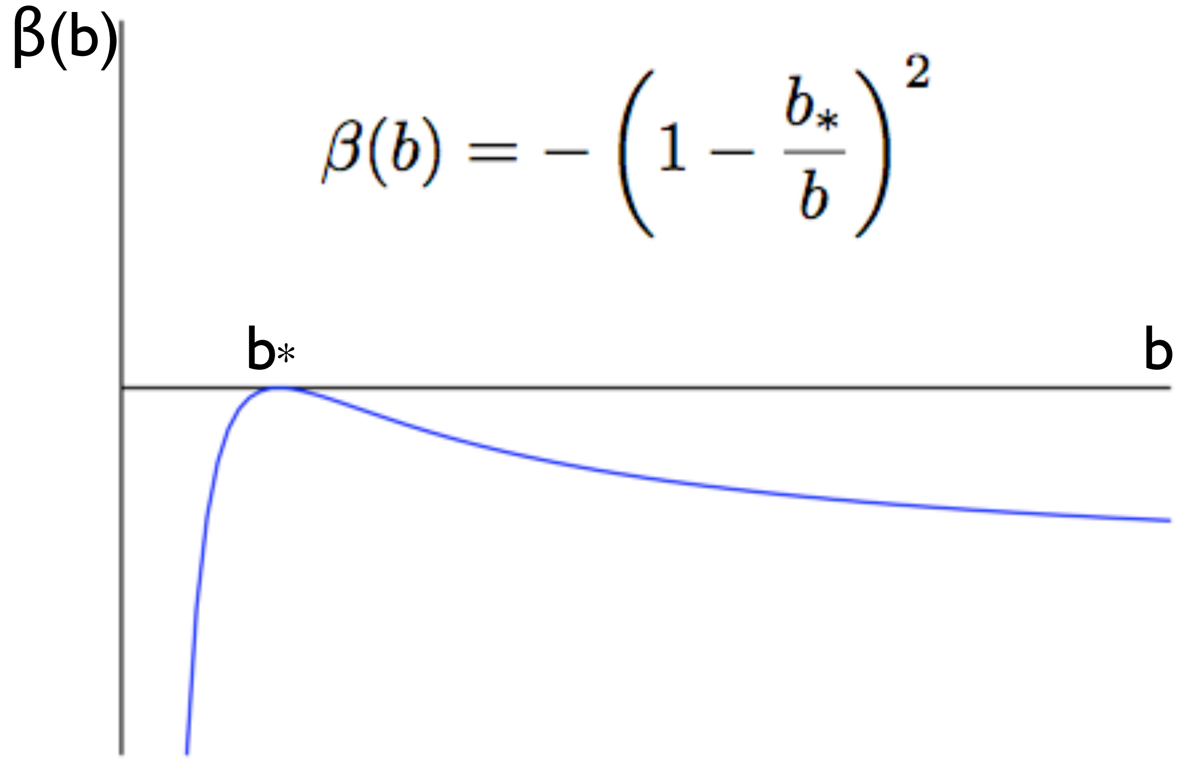

b
b_I
t

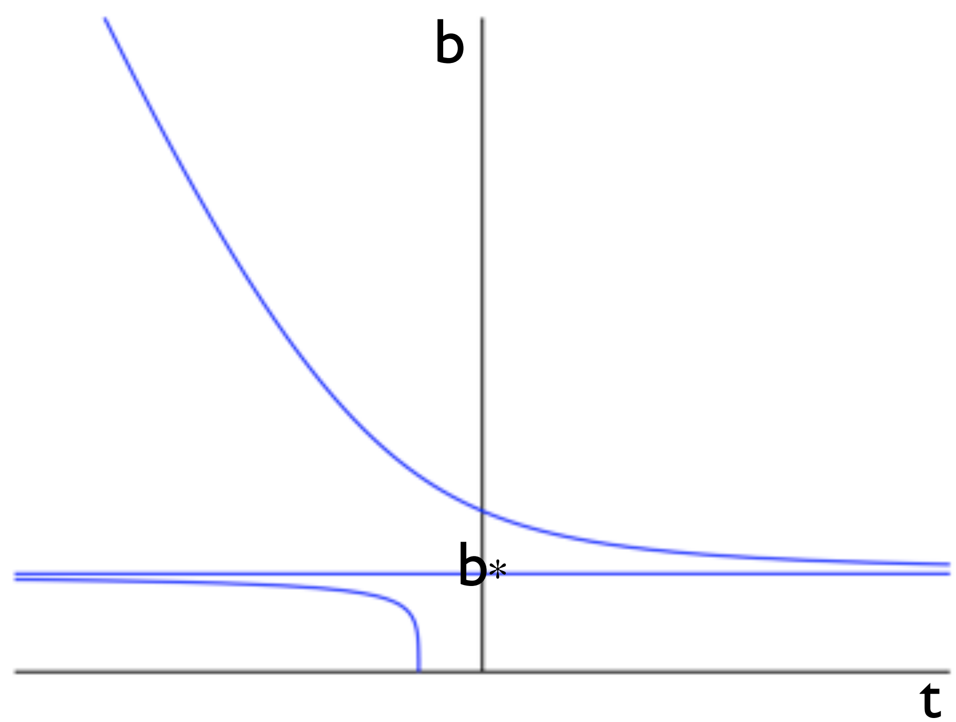


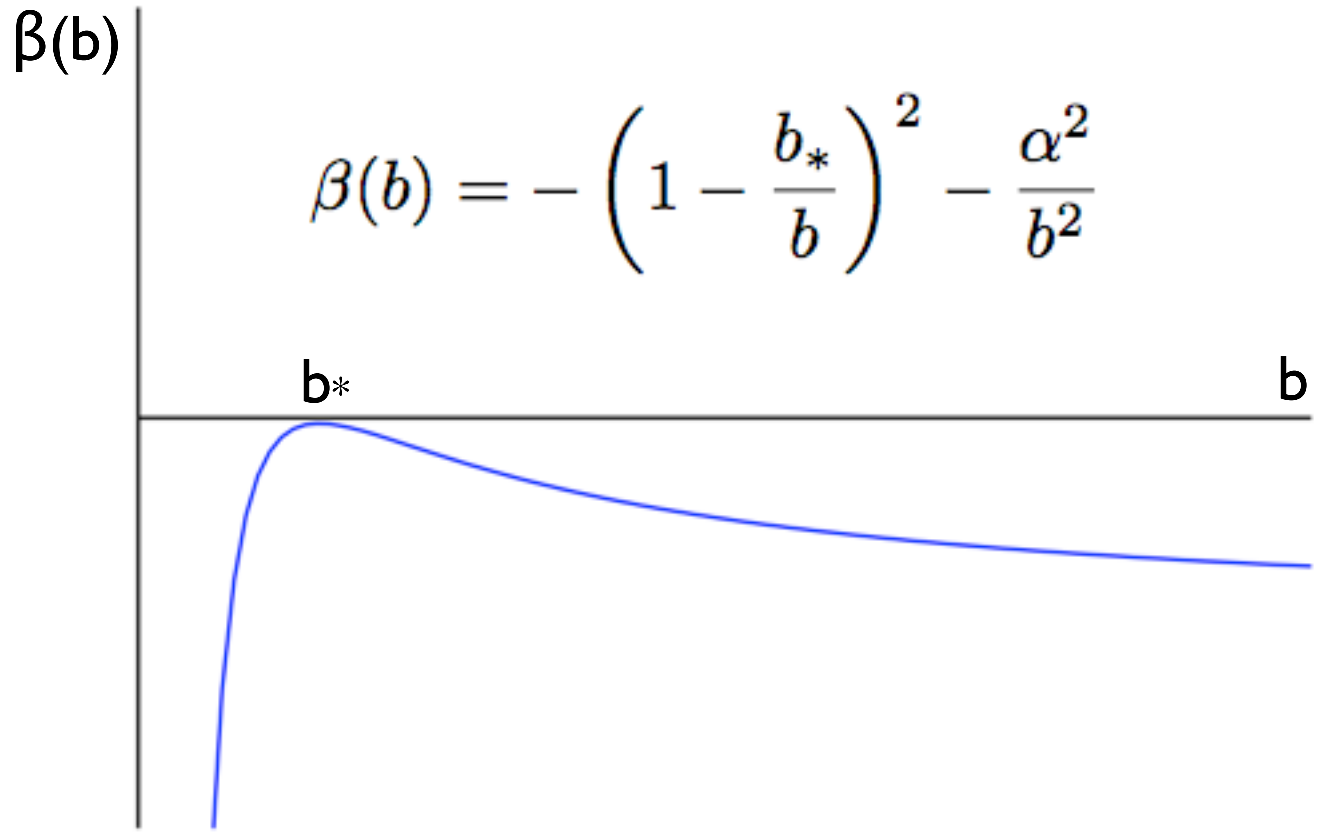
β(b)
$\beta(b) = -\left(1 - \frac{b_I}{b}\right)\left((1 - \frac{b_U}{b}\right)$
b_U
b_I
b

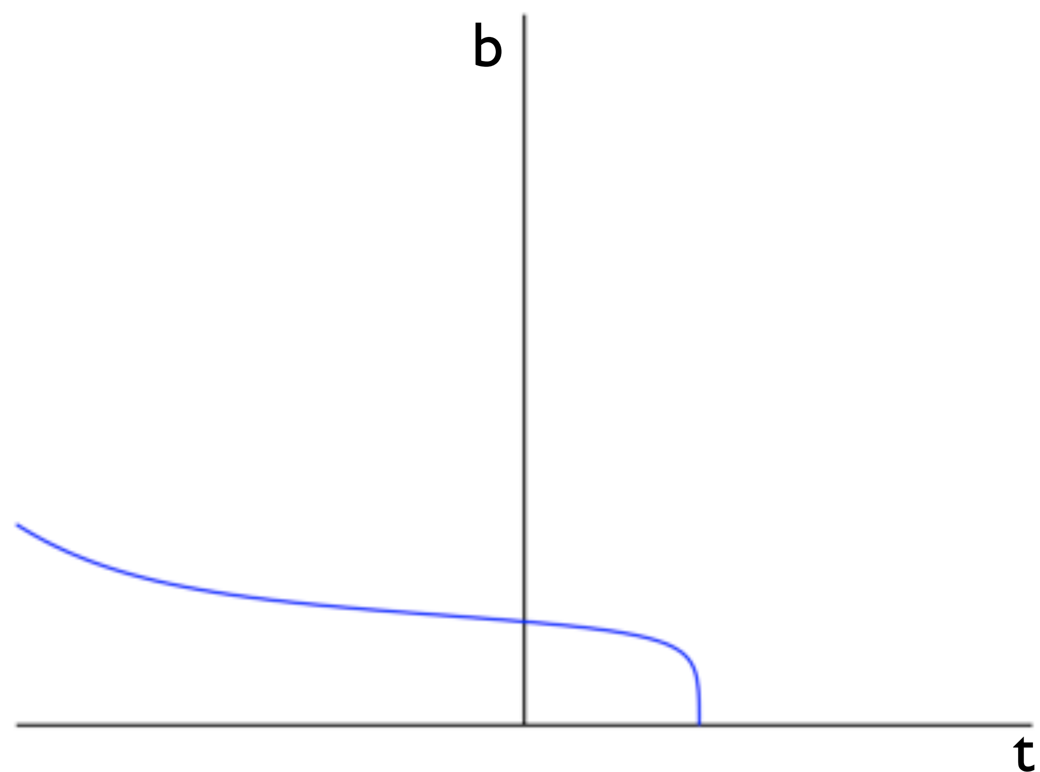


b
b_I
b_U
t

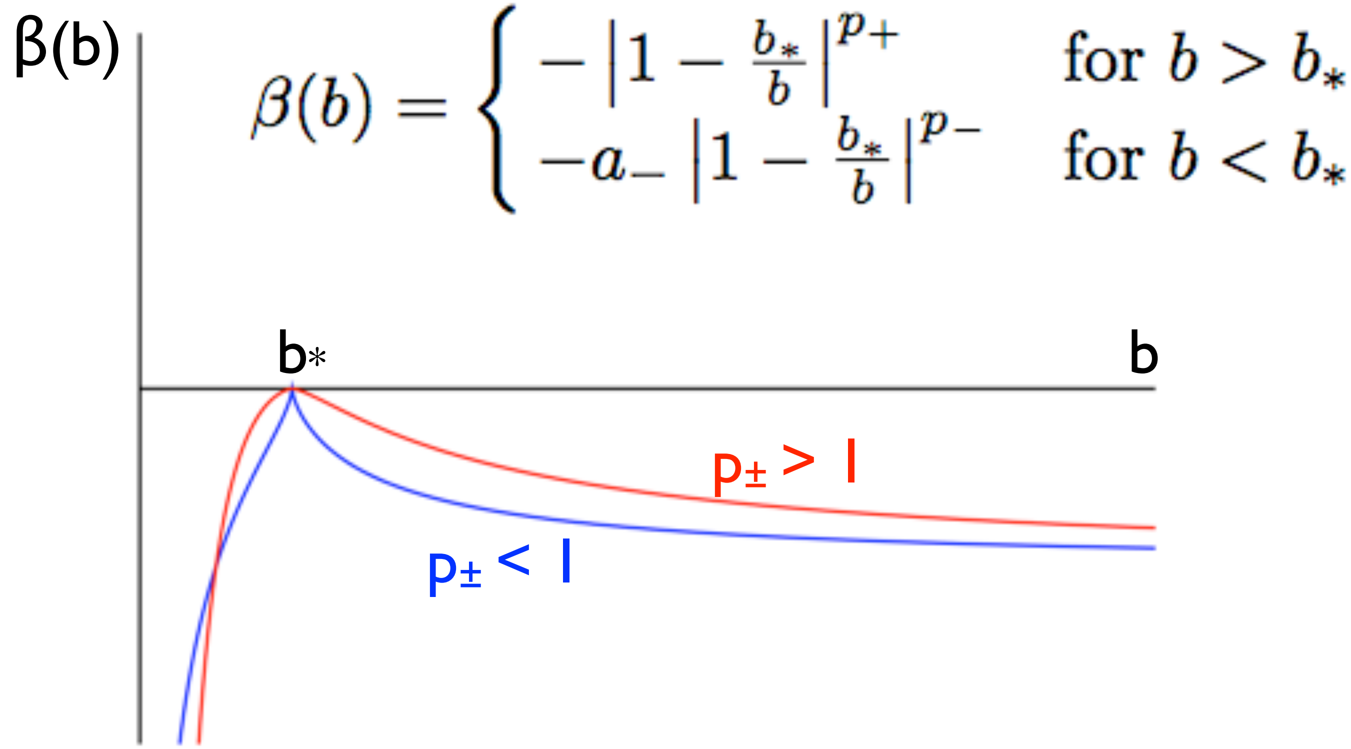

β(b)
$\beta(b) = -\left(1 - \frac{b_*}{b}\right)^2$
b*
b

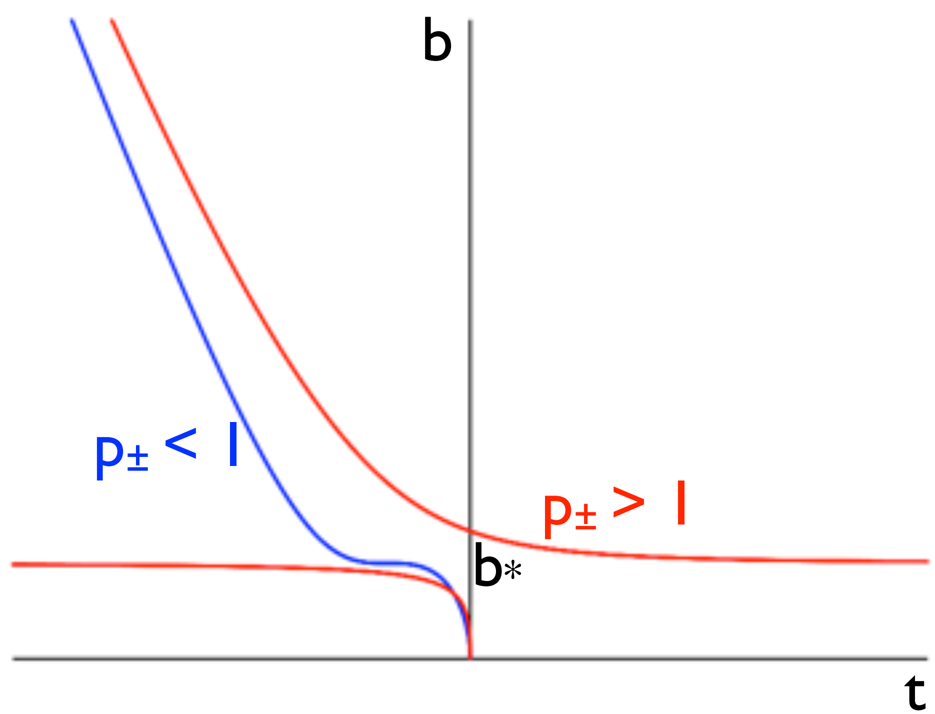

b
b*
t


β(b)
$\beta(b) = -\left(1 - \frac{b_*}{b}\right)^2 - \frac{\alpha^2}{b^2}$
b*
b


b
t

$$\beta(b) = \begin{cases} -\left|1 - \frac{b_*}{b}\right|^{p_+} & \text{for } b > b_* \\ -a_-\left|1 - \frac{b_*}{b}\right|^{p_-} & \text{for } b < b_* \end{cases}$$





Folded Wilson loop operators

An LXL Wilson loop operator is given by $W(L) = U_\mu^L U_\nu^L U_\mu^{\dagger L} U_\nu^{\dagger L}$

Eigenvalues of this operator are gauge invariant and lie on a unit circle in the complex plane

All eigenvalues will be close to +1 if the loop is small and there will be gap around -1

Eigenvalues will be distributed over the full unit circle for large loops

There is a critical size, $L_c(b,N)$, where the gap will close

We can define $a(b) = \dfrac{1}{L_c(b,N)}$ as a scale

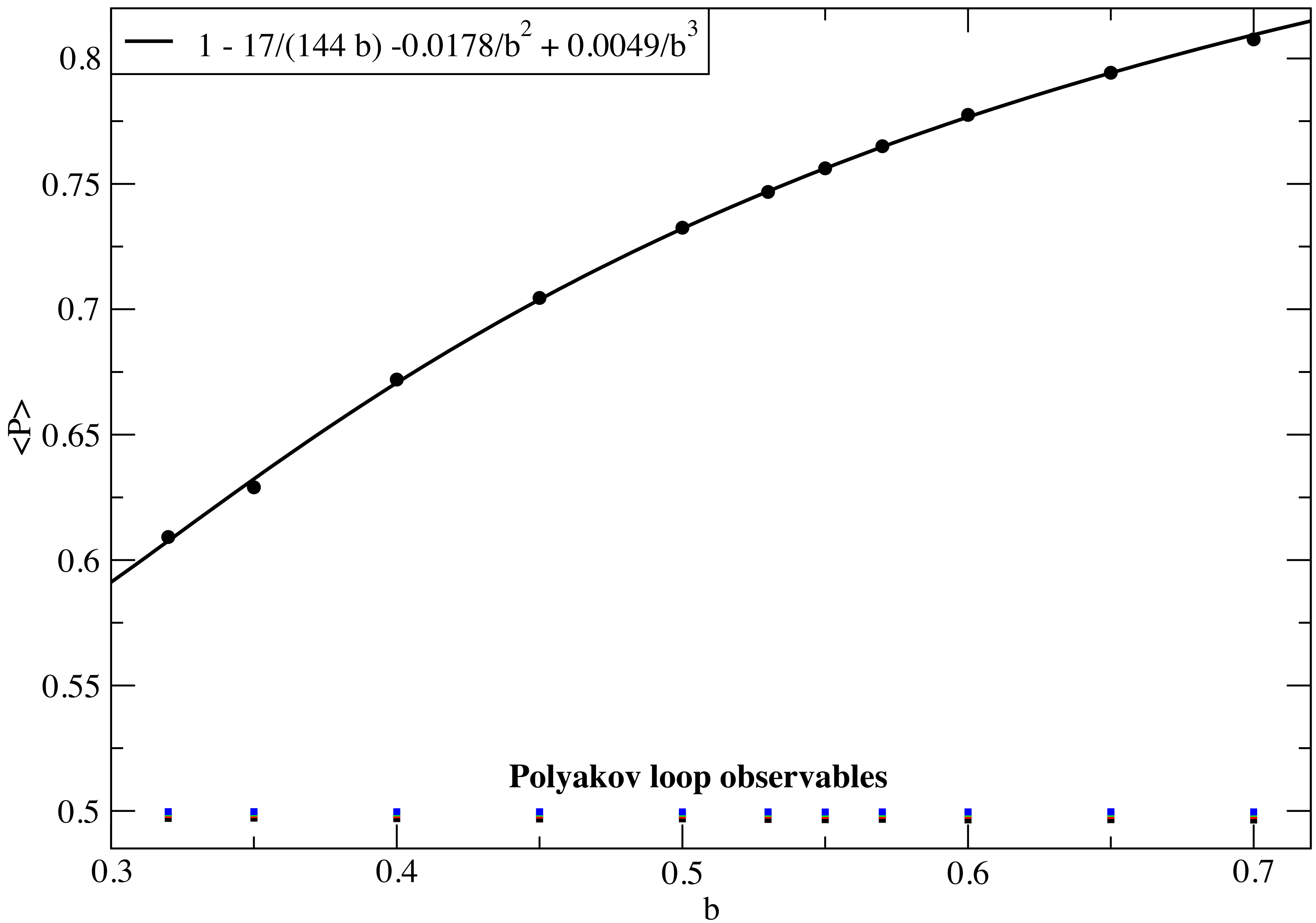
Eigenvalues of the Hermitian massless overlap Dirac operator

$\pm \lambda_k$ $\quad 0 < \lambda_k < 1,\ k = 1, \cdots, N^2 - 1$ with $\lambda_k < \lambda_{k+1}$

We can define $\lambda(b) = \langle \lambda_1 \rangle$ as another scale

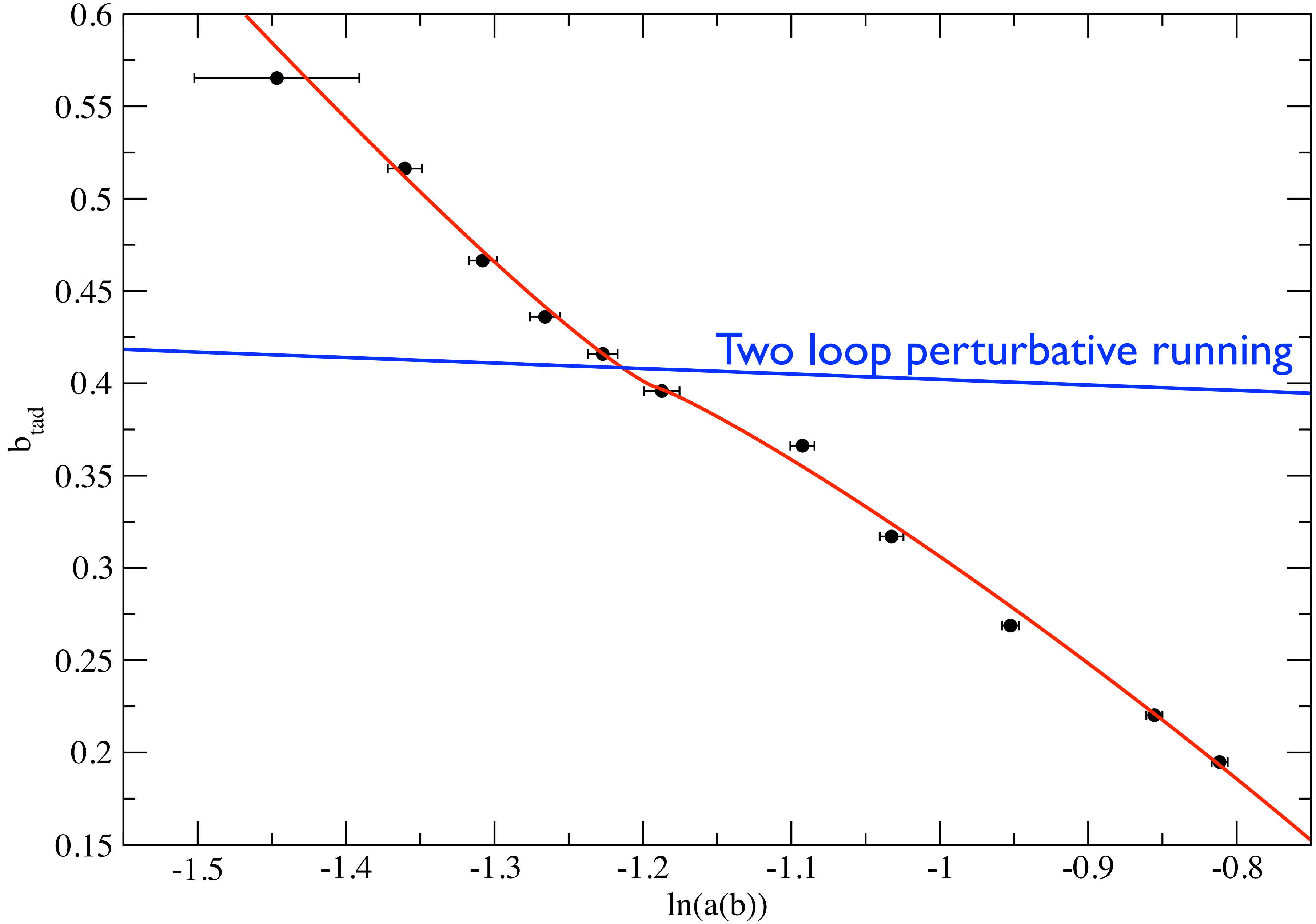
If chiral symmetry is broken, we expect the ratios $r_k = \left\langle \dfrac{\lambda_1}{\lambda_k} \right\rangle$ to be independent of coupling for small k at finite N

Average value of the plaquette

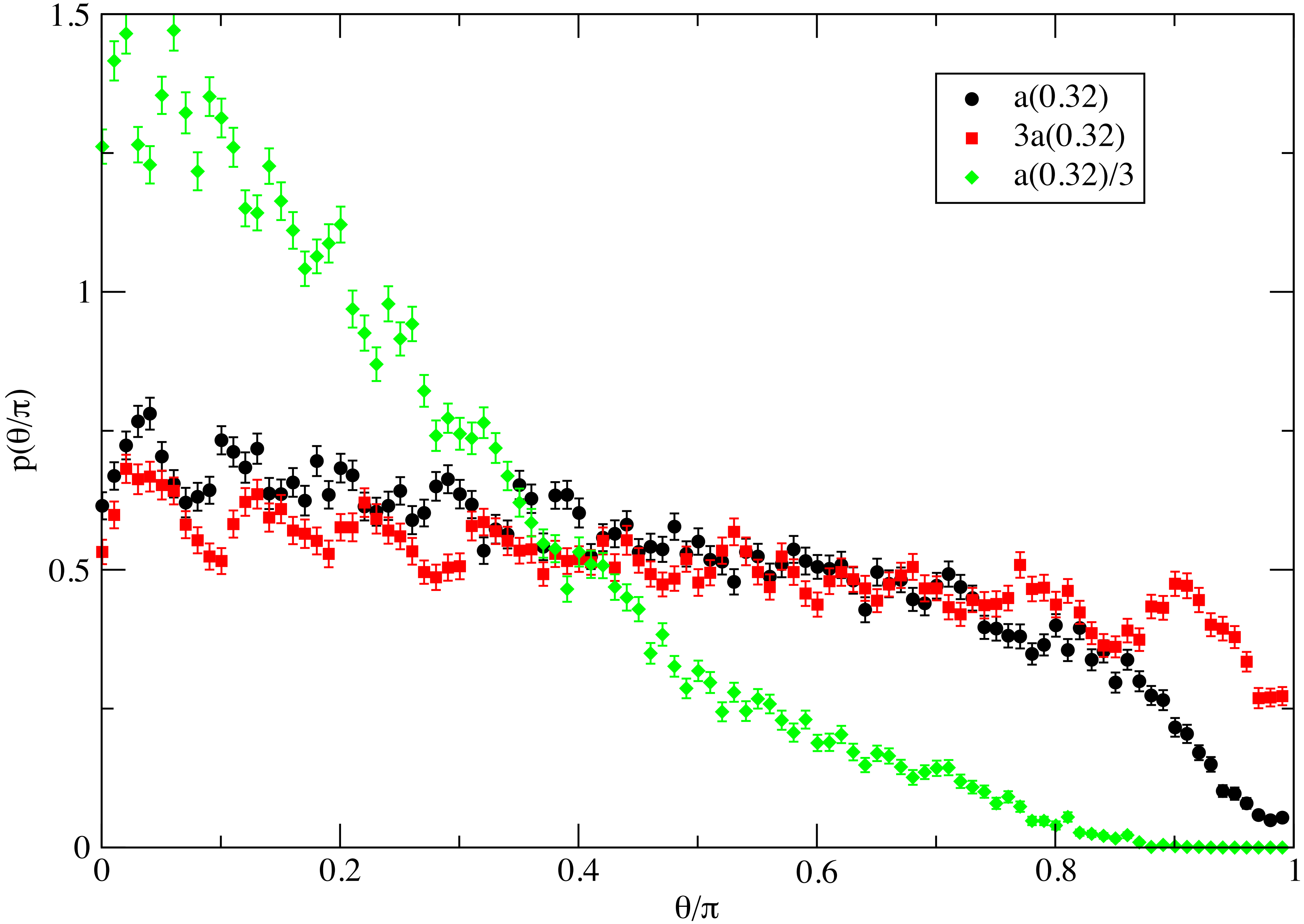


Polyakov loop observables show that we are in the continuum phase

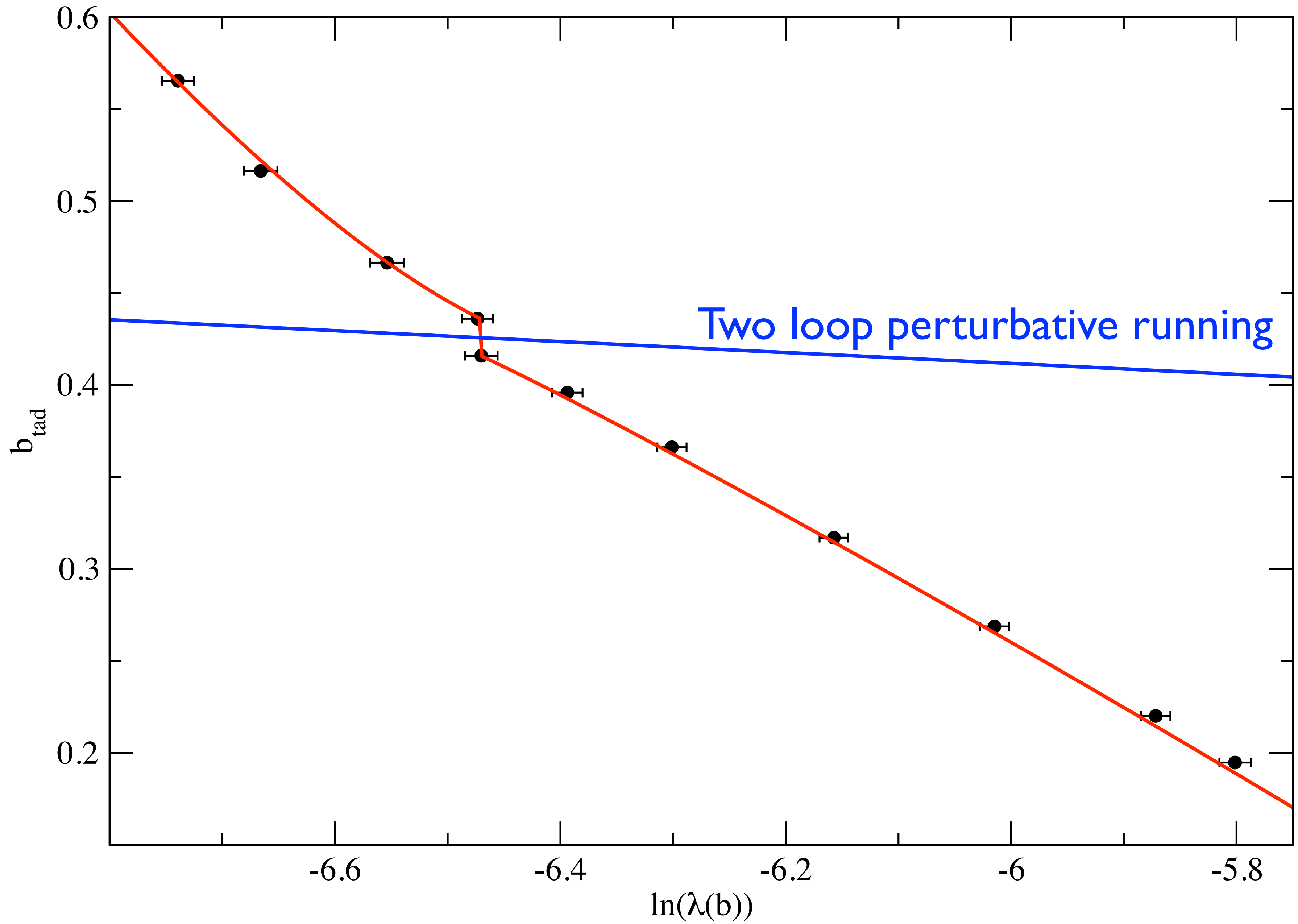
Running of the scale obtained from the folded Wilson loop operator



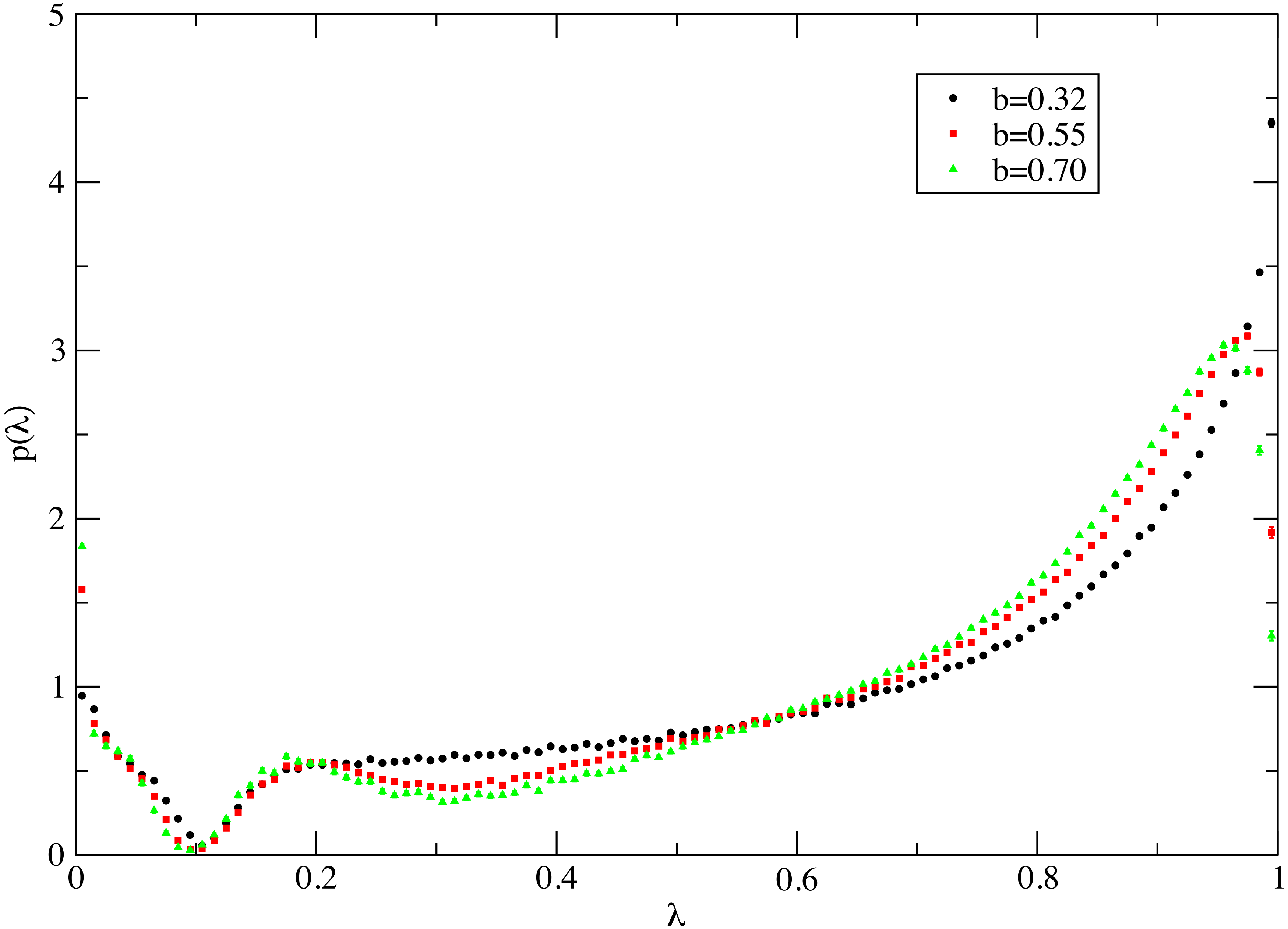
Distribution of the Wilson loop operator eigenvalues at , above and below criticality



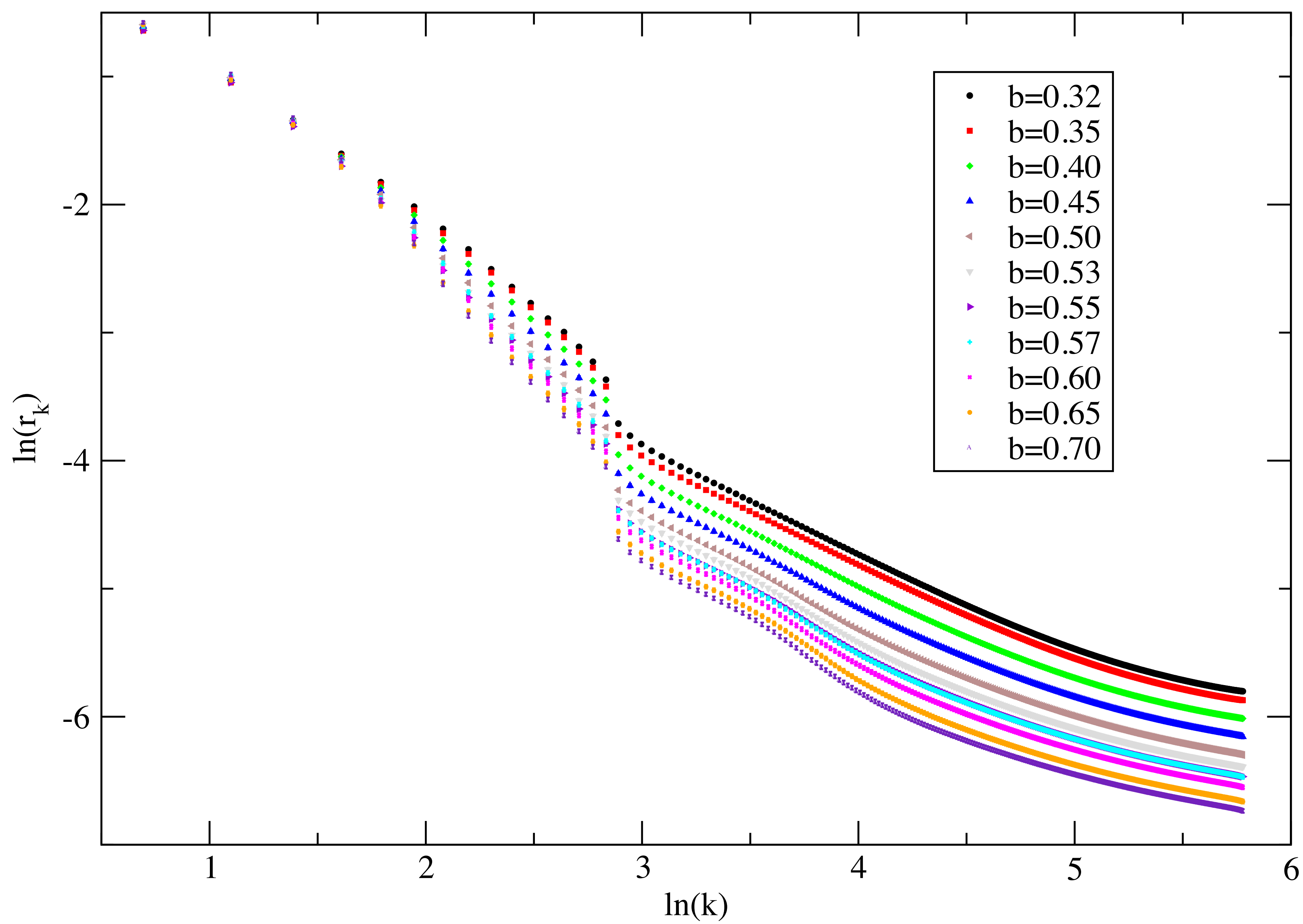
Running of the scale obtained from the lowest positive eigenvalue of the Dirac operator



Full distribution of the eigenvalues of the overlap Dirac operator at three different couplings



Ratios of the eigenvalues of the overlap Dirac operator



Results for the f=1 massless theory

using a(b) as the scale

$$\beta(b_{\mathrm{tad}}) = \begin{cases} -\left|1 - \frac{0.398}{b_{\mathrm{tad}}}\right|^{0.16} & \text{for } b_{\mathrm{tad}} > 0.398 \\ -0.64\left|1 - \frac{0.398}{b_{\mathrm{tad}}}\right|^{0.12} & \text{for } b_{\mathrm{tad}} < 0.398 \end{cases}$$

using λ(b) as the scale

$$\beta(b_{\mathrm{tad}}) = \begin{cases} -\left|1 - \frac{0.419}{b_{\mathrm{tad}}}\right|^{0.37} & \text{for } b_{\mathrm{tad}} > 0.419 \\ -0.36\left|1 - \frac{0.419}{b_{\mathrm{tad}}}\right|^{0.05} & \text{for } b_{\mathrm{tad}} < 0.419 \end{cases}$$

Combining the two results, we say that $b_* = 0.408(11)$ is a location of non-analyticity

All powers are less than unity and so we cannot define a continuum theory at b_* for the f=1 theory

Conclusions

- We can use matrix model in the large N limit to numerically study gauge theories coupled to adjoint fermions.
- We can look at conformal and near-conformal theories by extending f, the number of fermion flavors, to take on any real value.
- One can use exact HMC algorithm (compute the fermion force exactly) or use pseudo-fermion HMC algorithm.
- The beta function is expected to show non-analytical behavior at the infra-red/ultra-violet fixed point.
- It is possible to add a mass term and study the theory.